# Algebraic connection theory of L-modules

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# EPILOGUE

It is not clear how to carry over the covariant part of the theory as expounded in (8) - that is the part dealing with dual F-modules - to the corresponding L-case. The problem is that if we follow Nelson's definition of the dual of an F-module (see (8) p. 1) the dual of an L-module is itself not an L-module. Consider to that end an L-module $M$ and let $M'$ be the set of all module homomorphisms $\omega : M \to L$, denoted by $m \to \langle\omega,m\rangle$. It is obvious that $M'$ is a vector space but we cannot define a module composition $L \times M' \to M'$ by setting

$$\langle l\omega,m\rangle = l\langle\omega,m\rangle.$$

Nevertheless it is striking that many contravariant algebraic aspects of manifolds and especially of connections and curvature are not restricted to manifolds.
Another question raised by the theory is the following one: how far are differentiable functions and differentiable fields on a differentiable manifold dual to each other? It is clear that a certain form of duality between functions and fields exists since they have the structure of an F-module as well as of an L-module. The module compositions are $(f,X) \to fX$ respectively $(X,f) \to Xf$, where $f$ is a function and $X$ a field. As we have seen both structures generate analogous theories. The theory corresponding to the first case can be applied to manifolds but we do not know whether it is possible in the second case or not.